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# TECHNICAL NOTE

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## ON THE ELECTRON DENSITY DISTRIBUTION ABOVE THE F2 PEAK

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## SUMMARY

The distribution of free electrons in an isothermal upper ionosphere consisting of a binary ion mixture ( $O^+$  and  $H^+$ ) is discussed. It is shown that for diffusive equilibrium, which should prevail at altitudes well above the F2 peak, measurements of the electron density distribution permit the determination of other structural parameters of the upper atmosphere, such as the temperature and the concentration of oxygen ions and protons.



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## INTRODUCTION

The exploration of the ionosphere above the F2 peak has become possible only in recent years by means of rockets and satellites as well as by the ground-based incoherent backscatter technique. One of the basic quantities measured in these experiments is the electron density. At present only a limited number of electron density profiles above the F2 peak (extending to altitudes less than 1000 kilometers), have been obtained, while the higher altitude region is still virtually unexplored. With the anticipated extension of ionosphere rocket measurements to well beyond 1000 kilometers, it is of interest to consider a likely model of the electron density distribution above the F2 peak based on present knowledge.

## ALTITUDE DISTRIBUTION OF IONS IN DIFFUSIVE EQUILIBRIUM

It is generally agreed that at altitudes well above the F2 peak, the distribution of electrons and ions should be in diffusive equilibrium. While in a neutral atmosphere each constituent in diffusive equilibrium is distributed independently of the others, the distribution of an ionic species (especially a light one) in a heterogeneous ionosphere is influenced by the presence of the others because of the electric field resulting from the slight charge separation between electrons and positive ions, as pointed out by Mange (Reference 1).

In general, the distribution with height of an ionic constituent in diffusive equilibrium can be written as

$$n_i = n_{i0} \exp \left[ - \int_0^z \frac{\left( m_i - \frac{m_+}{2} \right) g}{kT} dz \right], \quad (1)$$

where the mean ionic mass is given by  $m_+ = \sum_i n_i m_i / \sum_i n_i$ , and where  $n_i$  is the number density and  $m_i$  the mass of the  $i^{\text{th}}$  ionic constituent,  $g$  is the acceleration of gravity,  $k$  is Boltzmann's constant and  $T$  is the absolute temperature in degrees Kelvin.

## EQUILIBRIUM DISTRIBUTION OF ELECTRONS IN THE UPPER IONOSPHERE

Because of charge neutrality (i.e., the number density of electrons equals the total number density of positive ions:  $N = \sum_i n_i$ ), the electrons are distributed according to a hypothetical species having the mean ionic mass  $m_+$ , and the electron density distribution is thus given by

$$N = N_0 \exp \left( - \int_0^z \frac{gm_+}{2kT} dz \right). \quad (2)$$

The same distribution is also obtained by simply assuming that ambipolar diffusion is the predominant process affecting the electron density distribution above the F2 peak. Computations by Shimazaki (Reference 2) show that diffusive equilibrium is established very quickly at altitudes above 300 kilometers.

The equilibrium electron density distribution is derived from

$$\text{div} (N \vec{v}_D) = 0. \quad (3)$$

In an isothermal atmosphere, in middle latitudes, the diffusion velocity  $\vec{v}_D$  is assumed to be acting mainly in the vertical ( $z$ ) direction as the result of gravity — although modified by the earth's magnetic field — and is given by

$$v_D = D_a \left( \frac{1}{N} \frac{\partial N}{\partial z} + \frac{m'}{m} \frac{1}{H} \right) \quad (4)$$

where  $D_a$  is the ambipolar diffusion coefficient which is directly proportional to  $\sin^2 I$  ( $I$  = magnetic dip) and inversely proportional to the atmospheric gas density;  $m'$  is the effective mass of the electron-ion gas;  $m$  is the mean molecular mass of the neutral gas within which the charged particles diffuse; and  $H$  is the neutral scale height.



The effective mass of the electron-ion gas is, in general, given by  $m' = m_+ T_i / (T_i + T_e)$ , where  $T_i$  and  $T_e$  are the ion and electron temperatures, respectively. For thermodynamic equilibrium,

$$m' = \frac{m_+}{2}.$$

Evaluating the condition of Equation 3 by using Equation 4, we obtain

$$\frac{1}{N} \frac{\partial N}{\partial z} = - \frac{m'}{m} \frac{1}{H} = - \frac{gm_+}{2kT} \quad (5)$$

which, after integration, is identical to Equation 2.

We shall now consider the electron-density distribution well above the F2 peak in an ionosphere consisting of a binary ion mixture ( $O^+$  and  $H^+$ ). According to satellite measurements (Reference 3),  $O^+$  is the predominant heavy ion in the upper ionosphere, while at greater altitudes the protons resulting from the telluric hydrogen corona become predominant (Reference 4).

The mean ionic mass in an isothermal  $O^+ - H^+$  ionosphere is given by

$$m_+ = \frac{m(H) \eta \exp Kz + m(O)}{1 + \eta \exp Kz}, \quad (6a)$$

where  $\eta = n(H^+)/n(O^+)$  at the reference level  $z = 0$ , and

$$K = \frac{[m(O) - m(H)]g}{kT}.$$

It is easily shown that

$$\int_0^z m_+ dz = \frac{1}{K} \left[ [m(H) - m(O)] \left\{ \ell_n (1 + \eta \exp Kz) - \ell_n (1 + \eta) \right\} + m(O) Kz \right]. \quad (6b)$$

Since  $\ln (1 + \eta)$  will be very small, we obtain for the electron density distribution according to Equation 2 or 5:

$$N = N_0 \exp \left[ - \frac{1}{2[m(O) - m(H)]} \left\{ [m(H) - m(O)] \ell_n (1 + \eta \exp Kz) + m(O) Kz \right\} \right]. \quad (7a)$$

For small  $z$ , Equation 7 reduces to

$$N \approx N_0 \exp \left[ - \frac{m(O) g z}{2kT} \right], \quad (7b)$$

while for large  $z$

$$N \approx N_0 \exp \left[ - \frac{m(H) g z}{2kT} \right]. \quad (7c)$$

It should be noted that at great altitudes (above 3000 kilometers) formula 7c is not strictly applicable because it neglects the earth's rotation. At these altitudes a modified hydrostatic equation including the centrifugal effects must be used. Equation 7c should correctly be expressed by

$$N(h) = A \exp \left[ \frac{g_0 m(H) R_0^2}{(R_0 + h) kT} + \frac{m(H) \Omega^2 (R_0 + h)^2 \cos^2 \theta}{2kT} \right], \quad (7d)$$

where

$h$  = the altitude,

$A$  = a normalization constant,

$g_0$  = the acceleration of gravity at the earth's surface,

$R_0$  = the earth's radius,

$m(H)$  = the protonic mass,

$\Omega$  = the angular velocity of the earth, and

$\theta$  = the latitude.

This equation, however, applies strictly only along magnetic field lines (Reference 4).

Recently, Hanson and Ortenburger have shown that there is only weak coupling between the upper F-region and the protonosphere due to the low diffusion rate of protons among oxygen ions (Reference 5). Thus, the protonosphere will not follow short-time (diurnal) variations in the upper ionosphere, a fact which is indicated by Whistler data.

As an illustration to the present discussion, Figure 1 shows electron-density profiles computed by means of Equation 7a for parametric values of temperature and proton-oxygen ion ratio at the reference level, which may suitably be chosen to be at about 500 kilometers. Also included is a profile based on rocket measurements by Berning (Reference 6), the only presently available experimental data up to 1500 kilometers. The experimental profile of Berning has been corrected for the maximum possible error at respective altitudes (Reference 6).

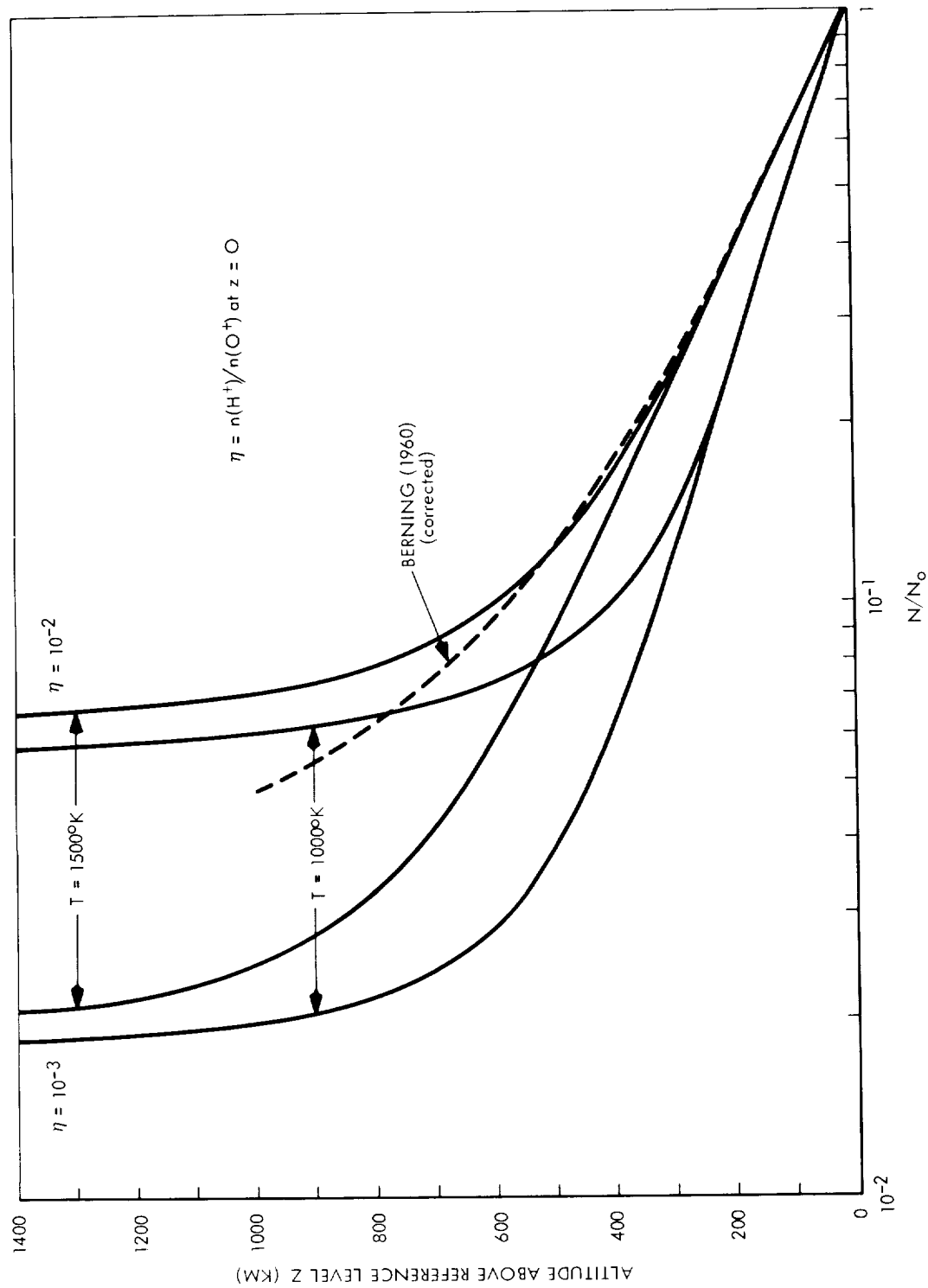


Figure 1 - Relative electron density distribution in an isothermal  $\text{O}^+ - \text{H}^+$  ionosphere

## CONCLUSION

It is obvious that from an observed electron density distribution well above the F2 peak the following atmospheric parameters can be deduced: (1) the scale height and the temperature from the portion of the N-profile with constant logarithmic slope (i.e., where  $O^+$  is predominant); and (2) the concentration of oxygen ions and protons, and the transition from the upper ionosphere to the protonosphere, from the curvature of the N-profile.

Thus, accurate measurements of the electron density distribution at altitudes well above the F2 peak represent an important tool for determining the structural parameters of the earth's outer atmosphere.

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